

1. **Eddy momentum fluxes.** Assume that a streamfunction field Ψ is defined:

$$\Psi = \sin(x + A y) \cos(y) - \sin(B y)$$

Where $\pi/2 \leq x \leq 2\pi + \pi/2$ and $-\pi/2 \leq y \leq \pi/2$. Also, A and B are constants. Note that $u = -\partial\Psi/\partial y$ and $v = \partial\Psi/\partial x$. To add additional interest, leave A and B as a variable in your solution and in your plotting program (be it FORTRAN, matlab, excel, etc.) Use circled values of A and B:

Let A = -0.9, -1.5, -0.5, -0.5, 0.5, 1.0, -0.3 and B = 1.2, 1.5, 1.5, 1., 1., 1.5, 1.5

a. Derive the general form of the zonal average of the eddy momentum flux: $[uv]$. Solve all integrals (it simplifies greatly). Hint: your result should only be a function of A and y.

b. (11 pts) Derive the formula for zonal mean eddy momentum convergence, $-d[uv]/dy$.

c. (4 pts) Make contour plots of Ψ and uv for the A and B specified over the x and y ranges given. Use 21 grid points in y and 41 grid points in x. Make sure axes and contour values are clearly labeled.

c. (5 pts) Make a single plot containing: $[\Psi]$, $[u]$, $[uv]$, and $d[uv]/dy$ for the A and B specified over the y range given. Use 21 grid points in y. Make sure axes and lines are clearly labeled.

2. **Kinetic energy maintenance.** Conceptually, zonal average momentum convergence should create zonal average westerly acceleration. However, plots of observed $[u'v']$ have maxima quite close to the observed maxima of $[u]$ leading to eddy momentum convergence ($\partial[u'v']/\partial\phi > 0$) far poleward of the $[u]$ maximum. Careful inspection of the KE equation finds cosine factors that bring the picture into consistency with intuition.

a. (2 pts) Imagine the distribution of vertical and time average $[u]$ is given by: $\{[u]\} = 20 \cdot \sin(2.9\phi - \pi/12)$ and that the distribution of the time and vertical average of $[u'v']$ is given by: $\{[u'v']\} = 22 \cdot \sin(2.9\phi - \pi/12)$. (Yes, nearly the same function.) Find the numeric values for $\{[u]\}$ and $\{[u'v']\}$ for the latitude range 10N to 70N. Use a 2.5 degree interval.

b. (2 pts) Find the numeric values of $\{[u]\}/\cos(\phi)$ and $\{[u'v']\} \cdot \cos^2(\phi)$ for 10N to 70N range with 2.5 degree interval.

c. (1 pt) Find $(-0.04) \cdot \{[u]\}/\cos(\phi) \cdot \partial(\{[u'v']\} \cdot \cos^2(\phi))/\partial\phi$ for the 12.5N to 67.5N latitude range (2.5 degree interval) using finite differences. Hint, ϕ is in radians in your finite difference.

d. (5 pts) On one chart, plot the 5 quantities calculated here in parts a, b, and c, for the 15N to 65N latitude range. Comment on the relative locations of the peak values of these 5 quantities. How well does momentum convergence match the inverse cosine-weighted zonal mean zonal flow? Make sure all aspects of your chart are clearly labeled.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.