

1. Isentropic mass weighted view versus a simple zonal average.

Let a perturbation pressure field be defined as: $P' = \sin(y)\{(1-z)\cos(x) - Qz\sin(x)\} - Syz$

Assume these quasi-geostrophic assumptions apply:

$\theta' = \partial P' / \partial z$ $v' = \partial P' / \partial x$ where primes denote nondimensional perturbations
dimensional (total) values are given by:

$\theta = \theta_s (1 + C * \theta')$ $P = P_s (1 + C * P')$ $v = 10 v'$ (in m/s) $Z = z * 10\text{km}$

The subscript “s” denotes a static state (dimensional) which is isothermal with temperature $T=260\text{K}$. The surface pressure is $P_s(0) = 1.e+5$ Pa. $C=0.02$. $Q=4$. $S=1$. And scale height $H (=RT/g)$ is constant.

a. (2 pts) Derive the formula for θ_s as a function of H , T_s , z and κ (where $\kappa = R/C_p$). P_s should not be in your final formula. Hint: $P_s = P_s(0) \exp(-Z/H)$ where if this Z is in km, then so must be H . Hint: you can check your answer: $\theta_s=265\text{K}$ is about 500m, $\theta_s=364$ is about 9km.

b. (4 pts) Derive the formulae for v' and θ' as general functions of (x, y, z) .

c. (6 pts) Make contour plots of: P' , v' , and θ' for $z=0$ and for $y = (0, \pi)$ and $x = (0, 2\pi)$. Do the same for $Z=9\text{km}$ ($z=0.9$). You should see a temperature profile consistent with a developing baroclinic wave and geostrophic meridional wind. Use 21 pts in x range; 11 pts in y range. Discuss how the P' trough location changes with height and relative to the θ' field.

d. (2 pts) Calculate a zonal average of v' at $(y, z) = (\pi/2, 0.9)$. You may solve this numerically using the 20 grid points in the zonal direction. Trapezoidal rule is ok. You can assume that the domain is periodic. Hint: don't double count the end point.

e. (4 pts) Derive the formula for the height of a potential temperature surface θ_0 . Test out your formula by plotting the height of the $\theta=265\text{K}$ surface for $x = (0, 2\pi)$ and $y = (0, \pi)$. Hint: your plot should look similar to fig. 3.18 if that figure ranged from -1 to 5 km (instead of 0-10 km).

f. (6 pts) Plot v' for $\theta=265\text{K}$ and $\theta=364\text{K}$ surfaces. Set the $v'=0$ for any point that is underground (i.e. having elevation <0).

g. (6 pts) Calculate 2 mass-weighted zonal averages at $y=\pi/2$ of v' : one along the $\theta=265\text{K}$ surface and one along the $\theta=364$. A simple mass weighting is to find the z value of each grid point used, then find the P_s of that z value, next multiply the v' value by the P_s value and then sum the corresponding values, call that the v' sum. Next calculate the corresponding sum for the P_s values of the grid points used. Finally divide the v' sum by the P_s sum. Note that you do not include any points in the sum that are underground. These upper and lower elevation mass-weighted averages will be consistent in sign with the poleward extension of a Hadley cell seen in θ coordinates.

NOTE: all homework is to be done by you as an INDIVIDUAL: no ‘group’ efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written. Full credit requires proper units be included. Any plot must be completely and unambiguously labeled, including title and axes. Show ALL math steps.