ATM 240 -- Winter 2006

Problem set #1 35 pts Due: 5 October 2006

1. (16 pts) Each of the items a) - d) below have 3 parts: (1) Write down a mathematical formula that expresses each of the following balances or physical concepts. (2) Be sure to define your symbols or variables unambiguously. For example, R might mean distance from axis of rotation, earth radius, gas constant, etc. make sure you have defined it for each equation. (3) You can get these from any source you wish, but please provide a specific reference of your source, including page number and equation number. If your source is the text, the reference might be: Grotjahn (1993, p. ###, eqn. #.#) for an example of the style – where # is a number.

a) ideal gas law b) hypsometric equation

c) hydrostatic balance d) angular momentum of an atmospheric air parcel

2. (10 pts) In data-sparse regions velocities are sometimes estimated from geostrophic winds. This simple problem attempts to estimate the error made in such an estimation for a zonal average. Assume that the true wind is given by the gradient wind formula. To simplify the mathematics, assume there is a single, semi-circular trough of width R in the middle of the domain. The trough has fixed, positive curvature. Otherwise the geopotential height contours (Z) are straight, oriented east-west, and decrease with increasing y. Also: (i) there is no meridional shear. (ii) contours of Z using 50 m interval are 196 km apart. (iii) $f = 10^{-4}\ s^{-1}$.

a) Find u_g

b) Find u where there is curvature (hint: it will be a constant there) for these curvature radii: 1000 km, 2000 km, and 4000 km.

c) Calculate the zonal averages: [u] and [u_g] for each of the three cases in part b. Then find the percent error, defined as: 100 ([u_g] – [u])/[u]

d) What latitude corresponds to this value of f (to 4 significant digits)?

3. (4 pts) conservation of mass

a) What is the total amount of mass in an atmospheric column of air extending from the surface to outer space? (The column has unit horizontal area, say, 1 m^2 .)

b) Deduce and write down a mathematical expression for conservation of mass for Earth's entire atmosphere. Be sure to define all terms you use. Incorporate spherical geometry.

4. (5 pts) No net east-west torque. Assume that the surface stress $\tau = C_d \rho |u| u$ in the zonal direction. A formula for no net torque (in the absence of mountain torques) might be:

$$\oiint_{surface} \tau . r^2 \cos^2 \varphi .. d\lambda .. r d\varphi = 0$$

where r is the radius of the earth, φ is latitude in radians, λ is longitude in radians, C_d is a drag coefficient, ρ is density, |u| is the magnitude of zonal wind: u. Let |τ| be a constant everywhere, but opposite sign in the tropics compared to both polar regions. Hence somewhere between the tropics and poles τ changes sign to satisfy no net torque over the Northern Hemisphere (NH). (Assume the N and S hemispheres are symmetric and so just do the N. Hemisphere.) What is that latitude to the nearest whole degree where τ changes sign? (Hint: it is OK to solve this problem graphically.) For full credit, include all work, starting with a general expression for total torque.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please.

ATM 240 – Fall 2006

Problem set #2 | 30 pts | Due: 17 October 2006

1. (16 pts) a. Convert the W/m^2 numbers in "Fig. 7" by Kiehl & Trenberth (1997) into percentages of 342 W/m^2. Prepare a table that compares your numbers with those given in figure 3.4. (Note: some numbers must be combined to make the comparisons.) Attached is the table you are to fill in.

b. Both datasets balance the net input and output, but they do it somewhat differently. Summarize in words how the two datasets disagree with emphasis upon how less in one quantity may mean more in another quantity in order to achieve the same balance.

2. (6 pts) Look over the paper: Different Data, Different General Circulations? A Comparison of Selected Fields in NCEP/DOE AMIP-II and ECMWF ERA-40 Reanalyses using the link from the course web page: e. Observing the General Circulaiton. Read the discussion in the text of the variable: _____________ with associated figure(s) on page(s) _______________ and summarize the differences in the form of a table. Your table should have two columns: left column is the region mentioned, and the right column is the corresponding numerical or 'qualitatively worded' difference mentioned in the text. Note that the difference must always be in terms of: **ERA-40 minus NCEP/DOE Reanalysis**. Put an overall title to your table of the variable(s) you are summarizing.

3. (8 pts) a. Derive the Stefan-Boltzmann law (6.03) from (6.01).

$$E = \sigma T^4 \qquad \text{Eqn: (6.03)}$$

It may be helpful to note that:

$$\int_0^\infty X^3 \left(\exp\{X\}-1\right)^{-1} dX = \frac{\pi^4}{15}$$

b. What is σ in terms of h, c, and k?

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

Comparison table for Radiation Quantities

Variable name	Grotjahn Fig. 3.4 value (% of incoming)	Kiehl & Trenberth comparable value (% of incoming)
Total incoming solar at top of atmosphere		
Total solar reflected by earth-atmos system (earth's total albedo)		
Solar radiation reflected by clouds		
Solar radiation reflected by atmosphere		
Solar radiation reflected by earth surface		
Solar radiation absorbed by atmosphere		
Solar radiation absorbed by earth surface		
Albedo of the atmosphere		
Albedo of the earth surface		
Surface longwave radiation passing through atmosphere to space		
Net surface longwave rad. Absorbed by atmosphere		
Atmospheric longwave emissivity deduced from surface emission		
Net longwave rad by atmosphere to space		
Net outgoing longwave radiation to space by earth-atmos system		
Surface sensible heat flux		
Surface latent heat flux		

ATM 240 – Fall 2006

Problem set #3 32 pts Due: 24 October 2006

1. Deducing a stream function picture of a mean meridional cell from the meridional velocity field in the 'tropics'. The meridional velocity field is given by these formulae:

$[v_u] = Um*\sin((500-P)b)*\cos(L(\varphi-\delta))$ for $100 \leq P \leq 500$

$[v_m] = 0$ for $500 < P < 700$

$[v_L] = Lm*\cos((1000-P)a)*\cos(L(\varphi-\delta))$ for $700 \leq P \leq 1000$

Where Um = 3 m/s, Lm = -4 m/s, $a = \pi/600$, $b = \pi/400$, $L = \pi/50$, $\delta=5$, P is in mb, and φ is in degrees latitude. The subscripts are only there to indicate sub-ranges of [v].

a. (5 pts) Print out these 110 values of [v]: a 5 degree latitude interval for the latitude range from 35N to 15S gives 11 different latitudes. A 100 mb interval for pressure ranging from 100 mb to 1000 mb gives 10 vertical levels. Arrange [v] in a table with clear indications of which axis is latitude, which is pressure and such that the appropriate value of P and φ for any element of your table can be easily deduced.

b. (4 pts) Make a contour plot of the values of [v] for the ranges listed in part a. Be sure that your axes and contour lines are unambiguously labeled.

c. (11 pts) Using the indefinite integral '(1)' in the handout, find streamfunction Ψ from this [v] field for the range of P and latitude in part a. Begin by deriving the 3 general formulae for Ψ: one for each sub range of pressure levels. Then print out the 110 corresponding values of the stream function similar to what you did in part a. Arrange Ψ in a table with clear indications of which axis is latitude, which is pressure and such that the appropriate value of P and φ for any element of your table can be easily deduced.

d. (4 pts) Make a contour plot of the values of Ψ for the ranges listed in part a. Be sure that your axes and contour lines are unambiguously labeled.

2. (8 pts) Estimating what level corresponds to the air temperature in the glass slab model. Using the handout for the "glass slab" model, perform these calculations. Let a_A=0.2852 (from Kiehl & Trenberth's "fig. 7"). Assume that the surface T_G = 288K (this is a rough average of data in fig 3.11 at 38N). Finally, assume that A_G = 0.97.

a. Use (6.06a) to estimate A_A = 0.xxxx (round off to 4 decimal places)

b. Find E_A

c. Find T_A

d. What range of pressure levels in both seasons shown in fig. 3.11 correspond to the T_A you found?

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

ATM 240 – Fall 2006

Problem set #4 30 pts Due: 31 October 2006

1. A simple linkage between the glass slab calculation, horizontal heat fluxes, and the motions of a winter "Hadley" cell. Rising motion occurs at 10S and sinking at 30N. Let $A_A = 0.85$, $A_G = 1.0$, $a_A = 0.3$. From figure 3.11a, b it may be assumed that T_G = 287 K at 30N, 299 K at 10S. Let T_A = 253 K at 30N; 263 K at 10S.

a. (8 pts) Find E_A and E_G, then find the E_S needed for radiative balance at each location using the glass slab model.

b. (2 pts) From figure 3.8a, one may estimate the observed solar radiation absorbed; call that E^*. At 30N E^*=180, at 10S let it = 310 W/m^2. The difference: E^* minus E_S is the amount of excess radiant energy per unit area. Call that Δ; and find that value for each location.

c. (5 pts) Δ is a loss of energy per second per square meter. What rate of heating averaged over the column is needed to balance this loss of energy if the surface pressure is 1.013×10^5 Pa? Assume that the heating rate (dT/dt) is a constant in the vertical. Note that force equals mass times acceleration, and that pressure is a force per unit area. Find this heating rate in K/day for both locations. Hint: C_P dT/dt has units of W/kg.

d. (5 pts) The dry adiabatic lapse rate is given by g/C_p where g=9.8 m/s^2 and C_p = 1004 J/(K kg). If the column of air is warming by sinking, what value of vertical velocity must be present at each location? For a bit of realism, assume that the air has lapse rate Γ = 6 K/km. Neglect horizontal advection. Express your answer in mm/s.

e. (2 pts) If the average updraft velocity in thunderstorms along 10S is 2 m/s and 95% of the updraft is balanced by local downdrafts, what fraction of the band along 10S is covered by thunderstorm updrafts?

f. (2 pts) From figure 3.21 the 1000 mb ht is 75 m at the equator and 130 m at 20N. Estimate the pressure gradient force at 10N. Assume the air has temperature 299K.

g. (3 pts) From fig. 3.29, the precipitation at 10S is 5 mm/day. Precipitation releases latent heat and that can be expressed first in W/m^2. Then express that latent heat as an average heating over the entire column of air (using part b as a guide) and express the heating rate first in K/s then K/day.

h. (3 pts) Assume the meridional wind is -2.6 m/s at 10N and 1000 mb elevation (based on fig. 3.16a). Estimate the frictional coefficient (α) if the pressure gradient force from part f is balanced by Coriolis force (U= -10m/s) and simple Rayleigh damping: $\alpha \rho v = PGF - \rho f U$. Compare this estimate with the low level coefficient used in figure 6.10b.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

ATM 240 – Fall 2006

Problem set #5 | 30 pts | Due: 7 November 2006

1. Momentum, eddy fluxes, and the Ferrel cell circulation.

a. (12 pts) Beginning with dM/dt = -R F_x (where M is defined on page 91 of the text) derive the following Gill formula:

$$\frac{[v]}{r}\frac{\partial[M]}{\partial\varphi} + [\omega]\frac{\partial[M]}{\partial p} = -r\cos(\varphi)\left\{\frac{1}{r\cos^2(\varphi)}\frac{\partial}{\partial\varphi}\left([u'v']\cos^2(\varphi)\right)+\frac{\partial}{\partial p}[u'\omega']\right\} - R[F_x]$$

Begin by writing out the 4 terms in the total derivative. Also write the scalar form of the continuity equation in spherical coordinates. Mountain torques are being ignored. Hints: you need to express the equation in flux form prior to applying the zonal and time averages. Notice that all equations must be in spherical coordinates.

b. (5pts) Estimate [M] at these latitudes: 20N, 30N, 35N, and 40N given zonal wind values of 25, 37, 35, and 28 respectively at those latitudes.

c. (3 pts) Derive a formula for ∂[M]/∂φ = A

d. (5 pts) Using observed values of [u] = 37 m/s at the jet stream axis, estimate A at that axis via a finite difference. Note that A has units of m^2 / (s rad). How does this value of ∂[M]/∂φ compare with making a finite difference using values at 20N and 40N obtained in part b?

e. (5 pts) Let [u'v'] = 28 m^2s^{-2} at 40N and 37 m^2s^{-2} at 30N (based upon fig. 4.12). Using values of [M] from part b at 30N and 40N, evaluate a centered finite difference of ∂[M]/∂φ at 35N. Then substitute that value in the Gill formula to estimate the [v] wind at 35N. Use a similar centered finite difference for the meridional derivative of [u'v']$\cos^2\varphi$ term on the right hand side. In this part, ignore all terms involving pressure (i.e. vertical) velocity and friction.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

ATM 240 – Fall 2006

Problem set #6 28 pts Due: 14 November 2006

1. A Carnot cycle model is applied to the Hadley cell circulation. Assume that the work done (per unit mass) by each air parcel as it completes one circuit of the Hadley cell is: $E = 1.4 \times 10^3$ J/kg. Assume that the air in circulation has these 4 rectangluar domains:

i). 35N - 25N from 200 - 1000mb
ii). 25N - 10N from 700 - 1000 mb
iii). 10N - 0N from 1000 - 200 mb
iv). 10N - 25N from 500 - 200 mb.

a. (5 pts) Calculate the total mass, M of air in circulation. (Sum the masses in the 4 domains.)

b. (3 pts) The flow is described by a piecewise continuous stream function field [Ψ] defined below. Note that these 4 pieces are each *trapezoidal* in shape, not rectangular. Where each trapezoid has common borders with adjacent trapezoids that are 'diagonal'. The stream function should match along each common border. Make a contour plot of [Ψ] in the meridional (φ =latitude and P =pressure) plane. Let B=3. for the purposes of this plot.

$[\Psi] = B * Q1$ where $Q1=(35-\varphi(deg))/10$ for: $35N>\varphi>25N$ and $200+Q1*300 < P < 1000-300*Q1$
$[\Psi] = B * Q2$ where $Q2=(1000-P(mb))/300$ for: $P > 700$ and $35-Q2*10 > \varphi > 10*Q2$
$[\Psi] = B * Q3$ where $Q3=\varphi(deg)/10$ for: $10 N >\varphi >0$ and $200+Q3*300 < P < 1000-300*Q3$
$[\Psi] = B * Q4$ where $Q4=(P(mb)-200)/300$ for: $P < 500$ and $35-Q4*10 > \varphi >10*Q4$

c. (5 pts)Using $B=3 \times 10^{11}$ Pa-m-s, find [v] and [ω] using (4.5) in the 4 areas defined in "b".

d. (6 pts) Let the middle contour of [Ψ] represent the average path taken by the various parcels. Calculate the time a parcel needs to complete one circuit of the Hadley cell following the middle contour of Ψ. This contour should follow these paths in THREE of the trapezoids:
#1: 350mb — > 850mb at 30N #2: 30N — > 5N at 850mb #4: 5N — > 30N at 350mb.
Finally, assume that the third trapezoid (850 — > 350mb at 5N) be traversed in 1 hour.

e. (2 pts) Calculate the surface area, A, of the earth covered by the cell (0N to 35N).

f. (4 pts) Find the total rate of energy release per unit area: M*E/(t*A). (Answer in W/m^2) Compare your answer to the daylong horizontal average radiation absorbed in this band of 328 W/m^2. Explain in your own words why there is a difference between absorption and energy release by the Hadley cell and why the difference has the size (small or large) that it has.

2. (3 pts) If thunderstorm updrafts occupy ½ of one percent of the area between 0 to 10 S during January and each thunderstorm has updrafts that cover an area of radius 6.5 km, how many thunderstorms are occurring in the region at any given moment?

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

ATM 240 – Fall 2006

Problem set #7 28 pts Due: 21 November 2006

1. Illustration of closed domain calculation of Ae to Ke conversion. Assume a closed domain is drawn which captures a low and a high at the surface. Half the domain includes the surface low (indicated by "L"). The other half of the domain includes the surface high (indicated by "H"). Each half has area 4×10^{12} m^2 . The range of P is 300 mb to 1000 mb. Assume that the volume average ω in each half has magnitude 2×10^{-3} mb/s for the developing low (1×10^{-3} mb/s for the decaying low); with rising motion above the surface low.

The 300 mb heights are as follows: 9250 m (H) and 9370 m (L) during the developing stage; 9350 m (H) and 9050 m (L) during the decaying stage.

The sea level pressures are as follows: 1018 mb (H) and 1008 mb (L) during the developing stage; 1018 mb and 1000 mb during the decaying stage.

a. (2 pts) Using a scale height assumption of H=8.3 km, find the number of meters elevation change for each change of 1 mb and round that number off to the nearest whole meter. Next, use the rounded number to convert the 4 surface pressures to 1000 mb heights above sea level.

b. (2 pts) Find the 4 values of TL and TH as defined on p. 137.

c. (4 pts) Calculate CKA for the developing and the decay stages using the last displayed formula on p. 137. Divide by g to get an energy conversion consistent with (4.36).

d. (2 pts) Assume that 6 developing and 6 decaying cyclones exist at any one time on the surface of the earth. First calculate the total amount of energy conversion (Ae to Ke) due to all frontal cyclones. Second, express this number as a rate of energy conversion per unit area (of the whole globe). The second number should have comparable magnitude to the energy conversions shown in Fig. 4.26. (Note: fig. 4.26 has units 10^5 J/m^2 for the energy reservoirs, and W/m^2 for the other items depicted.)

2. Using the following approximate technique, calculate ε during July at two locations south of South Africa: (20 E, 35 S) and (20 E, 45 S). Let the global mean surface pressure be 1013 mb; and global mean surface T be 285.

a. (1 pt) From this information calculate global mean surface Θ.

b. (2 pts) Next calculate surface Θ at the two locations using figs. 5.3 and 5.8.

c. (4 pts) Use the answers to the last step AND assume that the static stability over water is 2.2×10^{-4} K/Pa then calculate the pressure of the global mean surface Θ at each of the two locations.

d. (2 pts) Use the answers to the last step to calculate ε at both locations.

3. (10 pts) Prove the last displayed equation before (4.25) on page 120 by showing all steps to obtain the right hand side by starting with $\partial\theta/\partial p$. Apply the sigma overbar average as the last step.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

ATM 240 – Fall 2006

Problem set #8 32 pts Due: 30 November 2006

1. Theoretical models often use complex, nondimensional dependent variables as solutions. (See similar solution derived in §7.1). This problem illustrates how fluxes are distributed around an eddy of specific shape. The problem also gives practice working with complex variables. Hint: eddy fields must always be real-valued, but take the real part *after* applying derivatives in order to take advantage of complex variable math.

Let:

$$\Psi' = \mathbf{Re}\{ B(z) \sin(y) \exp(i (x-f*y)) \}$$

where: $0 \le x \le 2\pi$, $0 \le y \le \pi$, $0 \le z \le 1$. $B(z) = \cosh(2z) + a*\sinh(2z)$;
$a = - 1/(2c)$; c=cmplx(0.5,0.15) (c is a complex number); $f = 0.3$

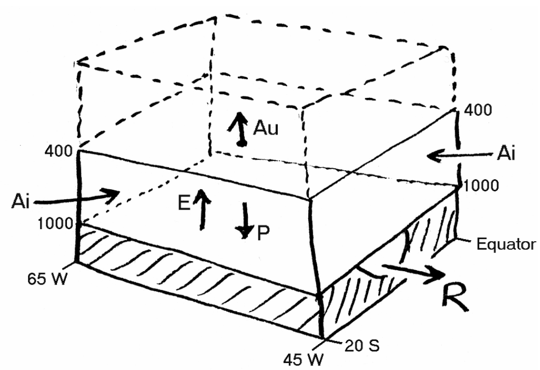
quasi-geostrophic relations apply: $u' = - \partial\Psi'/\partial y$ $v' = \partial\Psi'/\partial x$ $T' = \partial\Psi'/\partial z$.

a. (6 pts) obtain the formulas for the real parts of: u', v', and T' from the given Ψ' field.
b. (6 pts) make x vs y contour plots of these fields at the z=0.5 level: Ψ', T', u'v', v'T'.
c. (4 pts) make y vs z contour plots of the following averages over the range of x: [u'v'] and [v'T']. Use at least 11 grid points in z and 21 grid points in y.
d, (1 pt) make an x vs z contour plot of Ψ'

2. The Amazonian Hadley cell is more complex the closer one looks. This problem shows how some of the moisture is recycled.

The figure shows two layers to represent the atmosphere and one layer to represent the surface hydrology. Ai is the net inflow of water (vapor) mass in the lower layer. Au is the net outflow through the top of the lower layer into the upper layer. R is the Amazon river runoff. P is precipitation and E is evaporation. M is the horizontal area of the box. Observations show:

$Ai = 2\times10^8$ kg/s $R = 5.52\times10^{12}$ m^3/yr.
$P = 2.365$ m/yr $M = 5\times10^{12}$ m^2



a. (5 pts) Write down 2 formulas for water mass balance: one for the lower atmospheric layer, and one for the surface of the earth.
b. (4 pts) Find Au (units as Ai), and E (units as P). Find the amount of recycling, which equals the ratio of total evaporation divided by the runoff. (Hint: that ratio should be unitless.)
c. (6 pts) Set up and solve the equation for moisture flux Ae across the eastern wall given a velocity field u = 0.03 (P-600 mb)(cos(πφ/40) and mixing ratio q = 25 exp((P-1000mb)/200mb) where Pis in mb, φ is in degrees. Hints: your answer should have the units that Ai has, it will be larger than Ai.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.

Maybe in the future consider:

c. (6 pts) Estimate the flux through the eastern wall of the lower layer, given the following. (Define <s> to indicate the average value of s on the eastern wall.) Assume isothermal conditions with scale height H=8.15km. Let surface P = 10^5 Pa and dry air surface density = 1.25 kg/m^3. Mean mixing ratio <w> = 10 gm/kg. Mean zonal wind is <u>= - 2 m/s. The wall extends: 12S to 3N, 1000 to 400 mb. The density of liquid water, ρ_{water} =10^3 kg/m^3. Hints: find the height, Z_4 of 400 mb first; make your integral over Z and φ; the units should match Ai.

d. Alternatively, consider having a specified velocity field and moisture distribution and having students calculate the fluxes through the walls, to get an A_i

ATM 240 – Fall 2006

Problem set #9 22 pts Due: 7 December 2006

1. Tropical precipitation and divergence.

a. (3 pts) Find the energy production Q from precipitation over and area defined by: x ranging from 0 to 1000 km, y ranging from 0 to 1000km. The precipitation P = (5 cm/day) sin(πx/10^6m) sin(πy/10^6m) The energy production equals total precipitation mass times L the latent heat of vaporization. Units of Q are J/s.

b. (2 pts) Find the vertical average heating rate, q. Note that q = Q g/(C_p ΔP A). Here the average is over ΔP = 8x10^4 Pa, and A is the area of the domain. Units of q are K/s.

c. (3 pts) Assume that a portion of the heating causes a net increase in the temperature (=ΔT) at all levels. Since it is due to precipitation, it will have the same distribution: ΔT = (5 K) sin(πx/10^6m) sin(πy/10^6m). If the surface pressure is uniformly 1000 mb, and the elevation of the 200 mb surface (= Z_{200}) is initially uniformly 10 km. Find Z_{200} after ΔT is added to the initial temperature.

~~d. (5 pts) Assume the wind field is primarily in cyclostrophic balance. (Since this is the deep tropics, geostrophy is a poor approximation in part of this domain.) Find the general *functional* form of the components (u, v) in the domain; evaluate and combine constants. Assume it is a 'regular low' of the Northern Hemisphere. Then evaluate the zonal wind at y=1000, x=500. Hint: deduce the function form from locations where one component is zero.~~

e. (4 pts) If the divergence D = 2x10^{-5}) sin(πx/10^6m) sin(πy/10^6m). Find the functional form of the velocity potential, χ assuming all integration constants are zero. Then find the divergent wind, v_χ at x=500, y=1000.

f. (4 pts extra credit) Plot vectors of (u,v) on contours of Z_{200}. Plot vectors of (u_χ,v_χ) on contours of χ.

2. This problem relates the subtropical jet to the tropical divergent winds. The following *nondimensional* definitions reduce the complexity of the mathematics:

$$[u]\frac{\partial \varsigma'}{\partial x} = S = -v_\chi \frac{\partial}{\partial y}\left(f + [\varsigma]\right) - f D \quad where\, D = \nabla^2 \chi \ ,\ v_\chi = \frac{\partial \chi}{\partial y}\ ,\ [\varsigma] = \nabla^2 [\Psi]\ ,\ [u] = -\frac{\partial [\Psi]}{\partial y}$$

$$\chi = -A\sin(x)\cos(y)\ , \qquad [\Psi] = 1 - \frac{1}{4}\left\{ y - \cos(y) \right\}$$

Consider the ranges: x = (-π /2, 3 π /2), y = (0, π). Let A = 0.25 and f = sin(y/2).

a. (3 pts) Derive the functional form of S

b. (3 pts) Assuming that the perturbation vorticity is separable: ζ' = g(x) h(y). Find g and h.

c. (4 pts) Assume that total streamfunction Ψ = [Ψ] - ζ'. Produce unambiguously labeled contour plots of Ψ. Use at least 21 points in the x direction and 11 points in the y direction.

d. (2 pts extra credit) Plot contours of the zonal component of rotational wind based on Ψ.

NOTE: all homework is to be done by you as an INDIVIDUAL: no 'group' efforts, please. For written answers, please use a word processor, so that penmanship is not an issue. Equations and derivations can be *neatly* hand-written.