

1. (16 pts) Each of the items a) - d) below have 3 parts: (1) Write down a mathematical formula that expresses each of the following balances or physical concepts. (2) Be sure to define your symbols or variables unambiguously. For example, R might mean distance from axis of rotation, earth radius, gas constant, etc. make sure you have defined it for each equation. (3) You can get these from any source you feel convenient, but please provide a specific reference of your source, including page number and equation number. If your source is the text, the reference might be: Grotjahn (1993, p. XX, eqn. X.X) for an example of the style – where X is a number.

- ideal gas law
- hypsometric equation
- geostrophic wind balance in vector form
- angular momentum of an atmospheric air parcel

2. (4 pts) conservation of mass

- What is the total amount of mass in an atmospheric column of air extending from the surface to outer space? (The column has unit horizontal area, say, 1 m^2 .)
- Deduce and write down a mathematical expression for conservation of mass for Earth's entire atmosphere. Be sure to define all terms you use. Incorporate spherical geometry.

3. (5 pts) No net east-west torque. Assume that the surface stress $\tau = C_d \rho |u| u$ in the zonal direction. A formula for no net torque (in the absence of mountain torques) might be:

$$\oint_{\text{surface}} \tau \cdot r^2 \cos^2 \varphi \cdot d\lambda \cdot r d\varphi = 0$$

where r is the radius of the earth, φ is latitude in radians, λ is longitude in radians, C_d is a drag coefficient, ρ is density, $|u|$ is the magnitude of zonal wind: u . Let $|\tau|$ be a constant everywhere, but opposite sign in the tropics compared to both polar regions. Hence somewhere between the tropics and poles τ changes sign to satisfy no net torque over the northern hemisphere (NH). (Assume the N and S hemispheres are symmetric and so just do the N. Hemisphere.) What is that latitude to the nearest whole degree where τ changes sign? (Hint: it is OK to solve this problem graphically.)

4. (10 pts) Total daily insolation (Q_0) at the top of the atmosphere depends on latitude (ϕ), solar declination (δ), length of daylight (LOD) $24H/\pi$ in hours (where LOD in radians is $2H = 2 \arccos\{-\tan\phi \tan\delta\}$), and solar constant ($S = 1370 \text{ W/m}^2$). (See Physics of Climate, by Peixoto and Oort, 1992, p.99 for details). Let:

$$Q_0 = (24/\pi) S \{ H \sin\phi \sin\delta + \cos\phi \cos\delta \sin H \}$$

You will find the radiative equilibrium temperature from Q_0 for these situations: (i) equator at the equinox, (ii) 38°N at the equinox, (iii) 38°N at the summer solstice (23.5° degree declination) (iv) 38°N at the winter solstice (v) north pole at the summer solstice.

- Find $F = Q_0/24$ for the 5 situations listed.